

**Velocity and Other Rates of Change****Instantaneous Rate of Change**

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

**Example-Enlarging Circles**

a) Find the rate of change of the area  $A$  of a circle with respect to its radius  $r$ .

b) Evaluate the rate of change of  $A$  at  $r = 5$  and  $r = 10$ .

$$A = \pi r^2$$

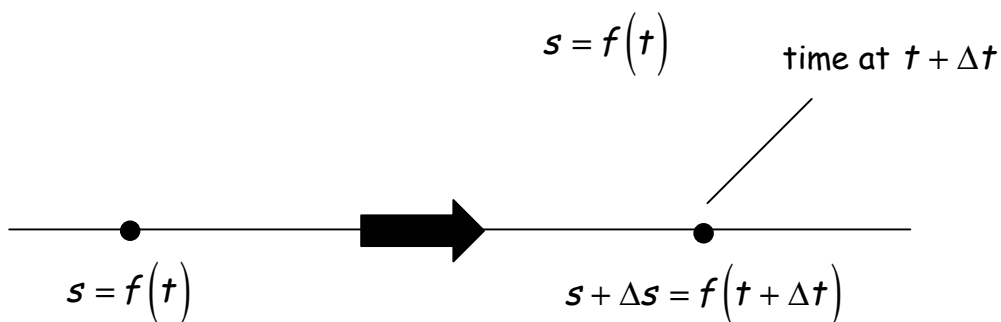
$$a) \frac{dA}{dr} = \frac{d}{dr}(\pi r^2) = \pi \cdot 2r = 2\pi r$$

$$b) r = 5 \rightarrow 2\pi(5) \approx 31.4$$

$$r = 10 \rightarrow 2\pi(10) \approx 62.8$$

**Motion along a line**

-Suppose that an object is moving across a line so we know that its position  $s$  along the line is a function of time  $t$ :



-The displacement of the object over the time interval from  $t$  to  $t + \Delta t$  is

$$\Delta s = f(t + \Delta t) - f(t)$$

-The average velocity of the object over that time interval is

$$v_{avg} = \frac{\text{displacement}}{\text{travel time}} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

### **Instantaneous Velocity**

-The instantaneous velocity is the derivative of the position function

$s = f(t)$  with respect to time

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

### **Speed**

-Speed is the absolute value of velocity

$$\text{Speed} = |v(t)| = \left| \frac{ds}{dt} \right|$$

### **Acceleration**

-Acceleration is the derivative of velocity with respect to time. If a bodies

velocity at time  $t$  is  $v(t) = \frac{ds}{dt}$ , then a bodies acceleration at time  $t$  is

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

**Example-Modeling a Vertical Motion**

-A blast propels a heavy rock straight up with a launch velocity of 160 ft/sec. It reaches a height of

$$s = 160t - 16t^2 \text{ feet after } t \text{ seconds}$$

a) How high does the rock go?

$$v = \frac{ds}{dt} = \frac{d}{dt}(160t - 16t^2) = 160 - 32t$$

-a rock has velocity 0 at the top so

$$160 - 32t = 0 \quad \text{or} \quad t = 5 \text{ sec}$$

-so the maximum height of a rock at  $t = 5 \text{ sec}$  is

$$160(5) - 16(5)^2 = 400 \text{ ft}$$

b) What is the velocity and speed of a rock when it is 256ft above the ground on the way up/down.

-To find the velocity when the height is 256ft determine the two values of  $t$  which  $s(t) = 256$

$$s(t) = 160t - 16t^2 = 256$$

$$16t^2 - 160t + 256 = 0$$

$$16(t^2 - 10t + 16) = 0$$

$$(t - 2)(t - 8) = 0$$

$$t = 2 \text{ sec} \quad \text{or} \quad t = 8 \text{ sec}$$

-The velocity at each of these times is

$$v(t) = 160 - 32t$$

$$v(2) = 160 - 32(2) = 96 \text{ ft/sec}$$

$$v(8) = 160 - 32(8) = -96 \text{ ft/sec}$$

-At both instances the speed is 96 ft/sec.

c) What is the acceleration of the rock at any time  $t$  during its flight?

$$a = \frac{dv}{dt} = \frac{d}{dt}(160 - 32t) = -32 \text{ ft/sec}^2$$

d) When does the rock hit the ground?

-The rock hits the ground at the positive time for which  $s = 0$ .

$$160t - 16t^2 = 0$$

$$-16t(t - 10) = 0$$

$$t = 0 \quad \text{or} \quad t = 10$$

-It was launched at  $t = 0$  so it hit the ground 10 seconds later.

**Example-Marginal Cost and Marginal Revenue**

Suppose it costs  $c(x) = x^3 - 6x^2 + 15x$  dollars to produce  $x$  radiators when 8 to 10 radiators are produced and that  $r(x) = x^3 - 3x^2 + 12x$  gives the dollar revenue from selling  $x$  radiators.

-You currently produce 10 radiators a day.

-Find the marginal cost and marginal revenue.

-The marginal cost of producing one more radiator when you are producing 10 is  $c'(10)$ .

$$c'(x) = \frac{d}{dx}(x^3 - 6x^2 + 15x) = 3x^2 - 12x + 15$$

$$c'(10) = 3(100) - 120 + 15 = 195 \text{ dollars}$$

-Marginal Revenue is:

$$r'(x) = \frac{d}{dx}(x^3 - 3x^2 + 12x) = 3x^2 - 6x + 12$$

$$r'(10) = 3(100) - 60 + 12 = 252 \text{ dollars}$$

-So it makes sense to produce another radiator!